SET CALCULUS

RECALL: Given two sets, A and B, we write:

- $A = B$ to mean $x \in A \iff x \in B$.
- $A \subseteq B$ to mean $x \in A \implies x \in B$.
- $A \cap B = \{x : x \in A \,\text{and}\, x \in B\}$. More generally, $\bigcap_{\alpha \in \Delta} A_\alpha = \{x : \forall \alpha \in \Delta,\, x \in A_\alpha\}$
- $A \cup B = \{x : x \in A \,\text{or}\, x \in B\}$. More generally, $\bigcup_{\alpha \in \Delta} A_\alpha = \{x : \exists \alpha \in \Delta,\, x \in A_\alpha\}$

To prove results about set operations, we'll make good use of propositional and predicate calculus!

EXAMPLE: Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Sketch a corresponding Venn Diagram.

GOAL: We need to show that $x \in A \cap (B \cup C) \iff x \in (A \cap B) \cup (A \cap C)$.

EXAMPLE: Prove $A \cap \bigcup_{\alpha \in \Delta} B_\alpha = \bigcup_{\alpha \in \Delta} A \cap B_\alpha$.

GOAL:

EXAMPLE: Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Sketch a corresponding Venn Diagram.

GOAL:

EXAMPLE: Prove $A \cup \bigcap_{\alpha \in \Delta} B_\alpha = \bigcap_{\alpha \in \Delta} A \cup B_\alpha$.

GOAL:

RECALL: Given two sets A and B, the **set difference** $A \backslash B = A - B = \{x : x \in A \text{ and } x \notin B\}$

NOTE: If X is a universal set and $A \subseteq X$, then $X \backslash A = X - A = \tilde{A}$ is the **complement** of A.

EXAMPLE: Prove $\tilde{\tilde{A}} = A$.

EXAMPLE: If $A, B \subseteq X$, show $A - B = A \cap \tilde{B}$.

EXAMPLE: Prove DeMorgan's Laws for Sets: $\widetilde{A \cup B} = \tilde{A} \cap \tilde{B}$. Sketch a corresponding Venn Diagram.

EXAMPLE: Use the last two proofs to prove: $\widetilde{A \cap B} = \tilde{A} \cup \tilde{B}$

EXAMPLE: Prove $\widetilde{\bigcup_{\alpha\in\Delta} A_\alpha} = \bigcap_{\alpha\in\Delta} \widetilde{A_\alpha}$

EXAMPLE: Prove $\widetilde{\bigcap_{\alpha\in\Delta} A_\alpha} = \bigcup_{\alpha\in\Delta} \widetilde{A_\alpha}$

EXAMPLE: What is $\bigcup_{\alpha\in\emptyset} A_\alpha$?

EXAMPLE: What is $\bigcap_{\alpha\in\emptyset} A_\alpha$?